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# Decoherence of two entangled atoms in vacuum

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## Abstract

Decoherence of two atoms initially prepared in Bell states in free space is discussed theoretically through calculation of the probabilities for the atoms to remain respectively in these states. It is found that, due to quantum interference, the atoms never follow a purely exponential law to decohere from their initial states, even though spontaneous emission is illustrated to cause the decoherence.

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## 1. Introduction

Any open quantum system inevitably interacts with its environment and changes the coherence existing among its constituents as a result. Such an effect is referred to as decoherence. Decoherence of various quantum systems is being studied largely for the reason that this process not only sheds light on the fundamental problem of where the possible boundary between the quantum world and the classical world lies [1], but also is important for the research in the theory of quantum information. The quantum information theory is well known to rely on correlations (entanglement) among qubits to achieve information processing that is otherwise impossible in the classical world [2]. But it is also well known that decoherence can reduce a system's entanglement and hence its efficiency of quantum information processing. In order to discover practical methods to suppress this unwanted decoherence, so that information can be processed without interruption, a thorough understanding of how decoherence takes place is evidently needed.

Presented in this work is a study of decoherence of two entangled atoms [3] that are stationary [4] in vacuum and interact indirectly with each other through vacuum fields. More specifically, the decoherence process is formulated via computing the probabilities for the atoms to remain in their initially entangled states. Compared with one commonly used method that depends first on a (nontrivial) master equation to determine the density operator of a system and then on the density operator to measure the entanglement of the system [5–7],

the present approach is not only simple in mathematics but also, more importantly, able to provide more direct information (not available in the preceding method) on how the system decoheres from its initially entangled states to other different entangled states. This latter point will become clear in the following discussion. Note also that, in contrast to other cases where two entangled atoms are placed in either a single cavity [8] or two isolated ones [5], more electromagnetic modes have to be considered in the present problem. Two-atom systems are frequently studied in the literature (not merely limited in the field of quantum information, of course) for the reason that they often allow exact solutions and can shed insight on more complicated systems.

For simplicity, the two atoms, A and B, are assumed to be identical and to have two states  $|E\rangle$  and  $|G\rangle$  whose energies are respectively  $\hbar\omega_E$  and  $\hbar\omega_G$  ( $\omega_0 \equiv \omega_E - \omega_G$ ). The location of atom A is chosen to be the origin of the Cartesian coordinate system used in the present study, and the location of atom B relative to the origin is chosen to be  $\vec{R} = R\hat{z}$  without loss of generality. Under these conditions, the interaction between the atoms and the vacuum can be represented by the following Hamiltonians in the minimal-coupling form [9]:

$$\begin{aligned} V_A &= \vec{\mu} \cdot \sum_s (\vec{\beta}_s a_s^\dagger + \vec{\beta}_s^* a_s) \\ &= \vec{\mu}_{GE} |G\rangle\langle E| \cdot \sum_s \vec{\beta}_s a_s^\dagger + \vec{\mu}_{EG} |E\rangle\langle G| \cdot \sum_s \vec{\beta}_s^* a_s \\ &\equiv V_A^\dagger + V_A^{-1}, \end{aligned} \quad (1)$$

$$\begin{aligned} V_B &= \vec{\mu} \cdot \sum_s (\vec{\beta}_s e^{i\vec{k}_s \cdot \vec{R}} a_s^\dagger + \vec{\beta}_s^* e^{-i\vec{k}_s \cdot \vec{R}} a_s) \\ &= \vec{\mu}_{GE} |G\rangle\langle E| \cdot \sum_s \vec{\beta}_s e^{i\vec{k}_s \cdot \vec{R}} a_s^\dagger + \vec{\mu}_{EG} |E\rangle\langle G| \cdot \sum_s \vec{\beta}_s^* e^{-i\vec{k}_s \cdot \vec{R}} a_s \\ &\equiv V_B^\dagger + V_B^{-1}, \end{aligned} \quad (2)$$

where  $\vec{\beta}_s = i\sqrt{2\pi\hbar\omega_0^2/(L^3\omega_s)}\vec{\epsilon}_s$  is the amplitude (containing a polarization unit vector  $\vec{\epsilon}_s$ ) of the  $s$ th quantized electromagnetic mode  $\omega_s$  ( $|\vec{k}_s| = \omega_s/c$ ) in the vacuum, and  $\vec{\mu}$  (assumed to be in the  $\hat{x} - \hat{z}$  plane) is the electric dipole moment operator of the atoms<sup>1</sup>. Those parts of the Hamiltonians in the preceding equations that contain field creation operators  $a_s^\dagger$  are denoted as  $V_i^\dagger$ , and those that contain field annihilation operators  $a_s$  are denoted as  $V_i^{-1}$ , where  $i = A, B$ . The symbol  $\vec{\mu}_{GE}$ , whose complex conjugate is  $\vec{\mu}_{EG}$ , is used to represent the matrix element of  $\vec{\mu}$  between  $|G\rangle$  and  $|E\rangle$ . Note counter-rotating terms are excluded in the interaction Hamiltonians, because virtual transitions supported by these terms are found to play merely a weak role in atomic evolution [11]. The total Hamiltonian of the present system that includes the two atoms and the vacuum is constructed as follows:

$$H = H_0 + V_A^\dagger + V_A^{-1} + V_B^\dagger + V_B^{-1}. \quad (3)$$

As usual, the unperturbed Hamiltonian  $H_0$  reads

$$H_0 = \underbrace{\hbar\omega_E |E\rangle\langle E| + \hbar\omega_G |G\rangle\langle G|}_{\text{atom A}} + \underbrace{\hbar\omega_E |E\rangle\langle E| + \hbar\omega_G |G\rangle\langle G|}_{\text{atom B}} + \sum_s \hbar\omega_s a_s^\dagger a_s. \quad (4)$$

<sup>1</sup> As in the reference cited in [10], the dipole moment operators of the atoms are assumed to be along the same direction for simplicity.

The zero-point energy of the vacuum is ignored in  $H$ , since this energy is constant and contributes little to the evolution of the system. Also ignored in  $H$  is the dipole–dipole interaction potential between the atoms; as a result, the separation  $R$  between the two atoms is required to satisfy the restriction  $R\omega_0/c \gg 1$  [12].

In the Schrödinger picture, the state of the system  $|\psi(t)\rangle$  at time  $t$  can be found from the system's initial state  $|\psi(0)\rangle$  with the help of the following integral [13]:

$$|\psi(t)\rangle = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} dz \frac{e^{-izt/\hbar}}{z - H} |\psi(0)\rangle, \quad (5)$$

where it is understood the denominator of the integrand contains an imaginary component  $i\eta$  ( $\eta \rightarrow 0^+$ ). This integral relation enables the decoherence process to be formulated in the present paper through tracing those atomic transitions that eventually reveal the time evolution of the initially entangled states, from which the probabilities for the system to be in these states are obtained. Since Bell states might be the best known and maximally entangled states [14] that exist between two identical atoms, the present paper is devoted to a specific case, in which the atoms are assumed to be initially in the (four) Bell states coupled to the vacuum states:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|E\rangle \otimes |G\rangle + |G\rangle \otimes |E\rangle) \otimes |0\rangle, \quad (6)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|E\rangle \otimes |G\rangle - |G\rangle \otimes |E\rangle) \otimes |0\rangle, \quad (7)$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|E\rangle \otimes |E\rangle + |G\rangle \otimes |G\rangle) \otimes |0\rangle, \quad (8)$$

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(|E\rangle \otimes |E\rangle - |G\rangle \otimes |G\rangle) \otimes |0\rangle. \quad (9)$$

Throughout the paper, in each tensor product of states, the left state is understood to represent the state of atom A, the middle state the state of atom B, and the right state the state of the field. Experimentally, Bell states have been constructed with two ions confined in a linear Paul trap and studied for their dynamic properties [15].

The remainder of the paper is organized as follows. The decoherence of the system from states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  and from  $|\psi_3\rangle$  and  $|\psi_4\rangle$  is studied in sections 2 and 3, respectively. In section 4, the results are summarized.

## 2. Decoherence from $|\psi_1\rangle$ and $|\psi_2\rangle$

Since states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  are all composed of two common components  $|E\rangle \otimes |G\rangle \otimes |0\rangle$  and  $|G\rangle \otimes |E\rangle \otimes |0\rangle$  (see equations (6) and (7)), decoherence properties of the system from these states can be determined via studying how these two components individually develop with time.

To formulate the time development of  $|\psi(0)\rangle = |E\rangle \otimes |G\rangle \otimes |0\rangle$ , which corresponds to an initial condition that atom A is in the excited state, atom B in the ground state, and no photons present, it is convenient to follow the procedure used in [11] to expand the Green function  $1/(z - H)$  in equation (5) into ascending powers of  $V_B$ ,

$$\frac{1}{z - H} = w_0 + w_0 V_B w_0 + w_0 V_B w_0 V_B w_0 + \dots, \quad (10)$$

where it is defined that

$$\begin{aligned}
 w_0 &= \left(1 - \frac{1}{z - H_0} V_A\right)^{-1} \frac{1}{z - H_0} \\
 &= \frac{1}{z - H_0} + \frac{1}{z - H_0} V_A \frac{1}{z - H_0} + \frac{1}{z - H_0} V_A \frac{1}{z - H_0} V_A \frac{1}{z - H_0} + \dots
 \end{aligned} \tag{11}$$

Consider operator  $w_0$  first. Obviously this operator only addresses the interaction of atom A with the vacuum and cannot modify the state of atom B. Since the first term in the serial expansion of  $w_0$  in equation (11) contains no interaction Hamiltonian  $V_A$ , it, when operated on  $|\psi(0)\rangle$ , merely represents one possibility that throughout the whole process atom A does not leave the excited state:

$$\frac{1}{z - H_0} |E\rangle \otimes |G\rangle \otimes |0\rangle = \frac{1}{z - E_0} |E\rangle \otimes |G\rangle \otimes |0\rangle, \tag{12}$$

where  $E_0 = \hbar(\omega_E + \omega_G)$ . In the second term, one  $V_A$  operator is included. Under the action of this operator, atom A can now transit from the excited state to the ground state and emit one photon:

$$\frac{1}{z - H_0} V_A \frac{1}{z - H_0} |E\rangle \otimes |G\rangle \otimes |0\rangle = \frac{1}{z - E_0} \sum_s \frac{A_s |G\rangle \otimes |G\rangle \otimes |1_s\rangle}{z - E_s}, \tag{13}$$

where  $E_s = 2\hbar\omega_G + \hbar\omega_s$ ,  $A_s = \vec{\mu}_{GE} \cdot \vec{\beta}_s$ , and the emitted photon can be in any mode  $|1_s\rangle$ . Note, in the third term, there are two  $V_A$  operators. For the structure of  $V_A$  and the present initial condition, the only possible event that can be brought about by this term is that the right  $V_A$  first causes atom A to transit to the ground state and to emit one photon, and the left  $V_A$  then immediately annihilates the photon just released and forces the atom to return to its excited state. Such an event is actually a radiation reaction process: the atom interacts with the photons it creates. Therefore, one has

$$\frac{1}{z - H_0} V_A \frac{1}{z - H_0} V_A \frac{1}{z - H_0} |E\rangle \otimes |G\rangle \otimes |0\rangle = \frac{E_A}{(z - E_0)^2} |E\rangle \otimes |G\rangle \otimes |0\rangle. \tag{14}$$

In the preceding equation,  $E_A = \sum_s \frac{|A_s|^2}{z - E_s}$ .

For the remaining terms on the right-hand side (RHS) of equation (11), it is recognized, following preceding arguments, that while those terms containing even (including 0) orders of  $V_A$  are only responsible for atom A to undertake repeated radiation reactions and stay in the excited state, those containing odd orders of  $V_A$  enable the atom first to perform repeated radiation reactions and then to transit to the ground state, accompanied by emitted photons. The action of  $w_0$  on  $|\psi(0)\rangle$  thus leaves the system in either its initial state or another state in which both the atoms are in state  $|G\rangle$  and one photon is created:

$$w_0 |E\rangle \otimes |G\rangle \otimes |0\rangle = \frac{|E\rangle \otimes |G\rangle \otimes |0\rangle}{z - E_0 - E_A} + \frac{1}{z - E_0 - E_A} \sum_s \frac{A_s |G\rangle \otimes |G\rangle \otimes |1_s\rangle}{z - E_s}. \tag{15}$$

By using the same method as that used in [11], one finds

$$E_A = -\frac{\Gamma_0 \hbar}{2\omega_0 \pi} \left[ \Omega + \omega_0 \ln \left( \frac{\Omega - \omega_0}{\omega_0} \right) \right] - i \frac{\Gamma_0 \hbar}{2},$$

where  $\Omega$  denotes the cut-off frequency of the vacuum modes, a parameter that is needed to make the nonrelativistic Hamiltonian  $H$  valid in the present discussion [16]. Note that  $E_A$  is identical to the corrected excited-state energy level of an isolated atom in vacuum and illustrates through its imaginary part the radiative instability of the atom's excited state [11]; this fact is expected, since, at this stage, atom B, although present, has no effect on atom A's transitions.

Thus, physically, under the influence of operator  $w_0$ , atom A is induced to decay spontaneously out of its excited state. In the preceding expression,  $\Gamma_0$  is the familiar spontaneous emission rate of an isolated excited atom in vacuum and reads  $\Gamma_0 = 4|\vec{\mu}_{GE}|^2\omega_0^3/(3\hbar c^3)$ .

Since the counter-rotating terms are ignored in the interaction Hamiltonians  $V_A$  and  $V_B$ , atom B, now in its ground state, can never jump into the excited state without first absorbing the photon emitted by atom A. Evidently, only the second term on the RHS of equation (15), which represents a photon is spontaneously emitted, allows the second operator  $w_0 V_B w_0$  on the RHS of equation (10) to achieve such a transition. Specific calculation yields

$$w_0 V_B w_0 |E\rangle \otimes |G\rangle \otimes |0\rangle = \frac{E_{AB}}{(z - E_0)(z - E_0 - E_A)} |G\rangle \otimes |E\rangle \otimes |0\rangle, \quad (16)$$

where  $E_{AB} = \sum_s \frac{A_s B_s^*}{z - E_s}$ , and  $B_s^*$  is the complex conjugate of  $B_s = \vec{\mu}_{GE} \cdot \vec{\beta}_s e^{i\vec{k}_s \cdot \vec{R}}$ .

While in the excited state, atom B has a chance to emit one photon and to return to the ground state; this process is realized with the help of the third operator  $w_0 V_B w_0 V_B w_0$  on the RHS of equation (10). But once atom B returns to the ground state, atom A, now in the ground state too, can have three different options: atom A can either ignore the photon emitted by atom B (see the first term on the RHS of equation (17)), repeatedly absorb and emit the photon and remain in the ground state (see the second term), or absorb the photon to go to the excited state and stay there (see the third term). Mathematically, these options are made possible by those components of the left  $w_0$  in the operator  $w_0 V_B w_0 V_B w_0$  that are respectively composed of  $1/(z - H_0)$ , even numbers of  $V_A$ , and odd numbers of  $V_A$ . Thus, the operator  $w_0 V_B w_0 V_B w_0$  leaves the system in three possible states:

$$\begin{aligned} w_0 V_B w_0 V_B w_0 |E\rangle \otimes |G\rangle \otimes |0\rangle &= \frac{E_{AB}}{(z - E_0)(z - E_0 - E_A)} \sum_s \frac{B_s |G\rangle \otimes |G\rangle \otimes |1_s\rangle}{z - E_s} \\ &+ \frac{|E_{AB}|^2}{(z - E_0)(z - E_0 - E_A)^2} \sum_s \frac{A_s |G\rangle \otimes |G\rangle \otimes |1_s\rangle}{z - E_s} \\ &+ \frac{|E_{AB}|^2}{(z - E_0)(z - E_0 - E_A)^2} |E\rangle \otimes |G\rangle \otimes |0\rangle. \end{aligned} \quad (17)$$

In general, it turns out that for a term that contains an even number of  $V_B$  operators in the serial expression of the Green function in equation (10), the allowed processes for atom B are repeated absorption and emission of the photons created by atom A, followed by a settlement of the atom in its ground state; for a term that contains an odd number of  $V_B$  operators, on the other hand, atom B can first execute the above-mentioned absorptions and emissions and then transit to the excited state. Between and after atom B's transitions, atom A can execute either pure photon absorptions and emissions or these actions plus transitions to the excited state. After all the terms on the RHS of equation (10) are accounted for, it follows that the system is only able to reside in three final states:  $|G\rangle \otimes |G\rangle \otimes |1_s\rangle$ ,  $|E\rangle \otimes |G\rangle \otimes |0\rangle$  and  $|G\rangle \otimes |E\rangle \otimes |0\rangle$ . This conclusion can be understood by noting that the present Hamiltonian, which is under the rotating-wave approximation, requires a photon to be created only when one atom transits from the excited state to the ground state and vice versa. These final states are also a representation that entanglement is formed among the atoms and fields [12]; note the initial state considered now is  $|\psi_0\rangle = |E\rangle \otimes |G\rangle \otimes |0\rangle$ , a separable state. The physical origin of such entanglement is that any of these atoms, if in the ground state, can have a chance to absorb the photons emitted by the other atom to go to the excited state, and becomes entangled with the other atom as a result. Since the first of the three final states is not needed in the calculation of the probabilities for the system to be in  $|\psi_1\rangle$  and  $|\psi_2\rangle$ , the following expansion

is expressed formally in terms of the rest two states alone:

$$\begin{aligned} \frac{1}{z-H}|E\rangle \otimes |G\rangle \otimes |0\rangle &= \frac{1}{2} \left( \frac{1}{z-E_0-E_A-|E_{AB}|} + \frac{1}{z-E_0-E_A+|E_{AB}|} \right) \\ &\times |E\rangle \otimes |G\rangle \otimes |0\rangle \\ &+ \frac{E_{AB}}{2|E_{AB}|} \left( \frac{1}{z-E_0-E_A-|E_{AB}|} - \frac{1}{z-E_0-E_A+|E_{AB}|} \right) \\ &\times |G\rangle \otimes |E\rangle \otimes |0\rangle, \end{aligned} \quad (18)$$

where  $E_B \equiv \sum_s \frac{|E_s|^2}{z-E_s} = E_A$  has been used.

After using the same method as that used in [11] and replacing  $z$  with  $E_0 + E_A$ , one finds that

$$\begin{aligned} E_{AB} &= \int_0^\Omega dx \left( a - x + i\frac{\Gamma_0}{2} \right) \frac{\Gamma_0/2}{(a-x)^2 + \Gamma_0^2/4} \left\{ \frac{3\hbar c}{2\pi\omega_0 R} \sin(xR/c) - \frac{3\hbar}{4\pi\omega_0 |\vec{\mu}_{GE}|^2} \right. \\ &\times \left[ \frac{|\mu_x|^2 \sin(Rx/c)}{xR/c} + (2|\mu_z|^2 - |\mu_x|^2) \left( \frac{\sin(xR/c)}{(xR/c)} + \frac{2\cos(xR/c)}{(xR/c)^2} \right. \right. \\ &\left. \left. - \frac{2\sin(xR/c)}{(xR/c)^3} \right) \right] \left. \right\}, \end{aligned} \quad (19)$$

where  $a = \omega_0 - \frac{\Gamma_0}{2\omega_0\pi} (\Omega + \omega_0 \ln \frac{\Omega - \omega_0}{\omega_0})$ , and  $\mu_x$  and  $\mu_z$  are respectively the  $x$ - and  $z$ -components of  $\vec{\mu}_{GE}$ . Since  $\Gamma_0/\omega_0 \sim 10^{-6}$  and  $\ln(\Omega/\omega_0) \sim 10$ , it is valid to introduce an approximation [13],

$$\frac{\Gamma_0/2}{(x-x_0)^2 + (\Gamma_0/2)^2} \simeq \pi \delta(x-x_0),$$

to carry out the integral in equation (19),

$$E_{AB} = i \frac{\Gamma_0 3\hbar a}{4\omega_0} \left[ \frac{\sin(Ra/c)}{(Ra/c)} \frac{|\mu_x|^2}{|\vec{\mu}_{GE}|^2} - \frac{2|\mu_z|^2 - |\mu_x|^2}{|\vec{\mu}_{GE}|^2} \frac{(Ra/c) \cos(Ra/c) - \sin(Ra/c)}{(Ra/c)^3} \right]. \quad (20)$$

The preceding approximation shows that of all the frequencies involved in equation (19), only the frequency  $a$  determined by the atoms' modified excited state and the ground state gives a dominant contribution to the integral. From the expression in equation (16), it is clear that  $E_{AB}$  can be interpreted as a parameter that measures the efficiency with which one atom absorbs photons emitted by the other atom, and, therefore, represents the strength of the interaction between the two atoms through radiation<sup>2</sup>. Note that as the denominators in equation (18) show, in addition to the radiation reactions, the energy level of the excited state is also changed by such atom–atom interaction. The expression in equation (20) indicates that unless the separation  $R$  between the atoms is infinite, the atom–atom interaction energy  $E_{AB}$  usually cannot be ignored. It is worth while to mention that the coupling between the two atoms is established through real photon exchanges as a result of the rotating-wave approximation made in  $H$ .

The method used to formulate the development of the initial state  $|E\rangle \otimes |G\rangle \otimes |0\rangle$  can also be applied to study that of another state  $|G\rangle \otimes |E\rangle \otimes |0\rangle$ , which corresponds to a different initial condition that atom A is in its ground state, atom B in the excited state, and no photons

<sup>2</sup> The atom–atom interaction considered here is evidently different from the well-known London interaction, which exists between two neutral atoms, as a result of the transient atomic polarization.

present. The only modification needed is that the Green function should be expanded now into a series in ascending powers of  $V_A$ :

$$\frac{1}{z-H} = w_1 + w_1 V_A w_1 + w_1 V_A w_1 V_A w_1 + \dots, \quad (21)$$

where

$$\begin{aligned} w_1 &= \left(1 - \frac{1}{z-H_0} V_B\right)^{-1} \frac{1}{z-H_0} \\ &\equiv \frac{1}{z-H_0} + \frac{1}{z-H_0} V_B \frac{1}{z-H_0} + \frac{1}{z-H_0} V_B \frac{1}{z-H_0} V_B \frac{1}{z-H_0} + \dots \end{aligned} \quad (22)$$

Following the same procedure as that used to derive the expression in equation (18), one gets

$$\begin{aligned} \frac{1}{z-H} |G\rangle \otimes |E\rangle \otimes |0\rangle &= \frac{1}{2} \left( \frac{1}{z-E_0-E_A-|E_{AB}|} + \frac{1}{z-E_0-E_A+|E_{AB}|} \right) \\ &\times |G\rangle \otimes |E\rangle \otimes |0\rangle \\ &+ \frac{E_{AB}^*}{2|E_{AB}|} \left( \frac{1}{z-E_0-E_A-|E_{AB}|} - \frac{1}{z-E_0-E_A+|E_{AB}|} \right) \\ &\times |E\rangle \otimes |G\rangle \otimes |0\rangle, \end{aligned} \quad (23)$$

where, as in equation (18), the components along  $|G\rangle \otimes |G\rangle \otimes |1_s\rangle$  have been ignored for the same reason. But, were the components ignored in equations (18) and (23) written explicitly, the relations in these equations, aided with those in equations (6) and (7), would have shown that the system actually evolves from its initially entangled states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  to other entangled states. Consequently, any measure of the entanglement derived from the density operator of the system can only give information about how the system's entanglement changes with time [5–7], not necessarily about how the system decoheres out of its initially entangled states. The present approach, which builds on finding the probabilities with which the system remains in its initially entangled states, can better address the latter decoherence process.

When the expressions in equations (18) and (23) are summed, substituted into equation (5), and projected on  $|\psi_1\rangle$ , the probability amplitude that the system remains in the initial state  $|\psi_1\rangle$  is obtained; see equation (6). To find the corresponding probability  $P_1$ , one simply takes the modulus square of the probability amplitude. Similarly, the probability  $P_2$  for the system to stay in state  $|\psi_2\rangle$  can also be found with the help of the relations in equations (18) and (23). It turns out that

$$P_1 = P_2 = \frac{e^{-t\Gamma_0}}{2} [1 + \cos(2t|E_{AB}|/\hbar)]. \quad (24)$$

The relation in equation (24) shows (through its exponential component) that spontaneous emission of each atom, which is a local decoherence process, is responsible for the non-local decoherence of the system [5]. But, the same relation also shows that, due to the atom–atom interaction energy  $E_{AB}$ , the non-local decoherence process of the system does not follow a purely exponential law (see the cosine function in the same equation). To understand the nonexponential behaviour of  $P_1$  and  $P_2$ , it is necessary to note two points. First, as the discussion presented in this section already reveals, see equations (15) and (17) for example, atom B in the ground state can eliminate the photon created by atom A to transit to the excited state, and, when atom B later returns to the ground state, it can, via the photon it emits itself, force atom A to transit back to the excited state with a nonzero probability. Thus,



spontaneous emission of one excited atom is actually influenced by the presence of the other atom and can no longer be a local process any more [10]. Second, a close examination of the expressions in equations (18) and (23) additionally exhibits that the atom–atom interaction splits the excited energy level of each atom (already modified by the radiation reactions) into two sublevels  $E_A + |E_{AB}|$  and  $E_A - |E_{AB}|$  and effectively changes the two two-level atoms into two multi-level atoms. The cosine function in equation (24) is produced as a result of quantum interference between the atomic transitions from these shifted excited states to the ground state; it is this quantum interference that deprives the system of its coherence at a period of  $(2N + 1)\pi\hbar/(2|E_{AB}|)$ , where  $N$  is any nonnegative integer. Quantum interference in a multi-level atomic system can bring about other different effects, one of which is the modification of spontaneous emission spectrum [17].

Equations (18) and (23) also show that the entangled states  $|\psi_1\rangle$  and  $|\psi_2\rangle$  remain pure states even suffering the atom–field interaction. This is because, although the vacuum fields have many modes, only a small fraction of these modes in a narrow range of the resonance frequency  $a$  are efficiently coupled to the atoms.

### 3. Decoherence of $|\psi_3\rangle$ and $|\psi_4\rangle$

For the Hamiltonian  $H$  considered in the present work (the counter-rotating terms excluded), the system, if initially in the state  $|G\rangle \otimes |G\rangle \otimes |0\rangle$ , will remain in that state permanently,

$$\frac{1}{z - H} |G\rangle \otimes |G\rangle \otimes |0\rangle = \frac{1}{z - 2\hbar\omega_G} |G\rangle \otimes |G\rangle \otimes |0\rangle, \quad (25)$$

and the time evolution of the entangled states  $|\psi_3\rangle$  and  $|\psi_4\rangle$  given respectively in equations (8) and (9) is only determined by that of the component  $|E\rangle \otimes |E\rangle \otimes |0\rangle$ . But, unlike the cases discussed in the last section, where excitation of one atom from the ground state to the excited state relies entirely on the other atom's release of one photon in the opposite transition, the atoms in state  $|E\rangle \otimes |E\rangle \otimes |0\rangle$  are now all initially in the excited state and, thus, can perform independent transitions. The atoms can, of course, interact with each other. For example, they can first transit from the excited state to the ground state and then absorb the photons emitted by the other to return to the excited state. But, since such a process is proportional to  $A_s B_t A_t^* B_s^*$ —three orders smaller than the leading terms in the above discussion (see, for example, equation (13)), it can be practically ignored for its negligible magnitude. The ignorance of the atom–atom interaction (still through the vacuum) in this section is also supported by the restriction imposed on the system in section 1:  $R\omega_0/c \gg 1$ , which significantly suppresses the communications between the atoms. To make the following mathematical analysis as simple as possible, an independent-transition approximation is assumed.

After the Green function is expanded as in equation (10) and a procedure similar to that used in section 2 is followed, one finds

$$\frac{1}{z - H} |E\rangle \otimes |E\rangle \otimes |0\rangle = \frac{|E\rangle \otimes |E\rangle \otimes |0\rangle}{z - 2\hbar\omega_E - E'_A - E'_B}, \quad (26)$$

where other terms, although illustrating that the system can evolve into different entangled states, are omitted for their irrelevance in the determination of the probabilities  $P_3$  and  $P_4$  that the system remain, respectively, in  $|\psi_3\rangle$  and  $|\psi_4\rangle$ . Still in equation (26),  $E'_A = \sum_s \frac{|A_s|^2}{z - \hbar\omega_G - \hbar\omega_E - \hbar\omega_s} = E'_B$  and can be evaluated by following the same method used to calculate  $E_A$ , provided  $z$  is now replaced by  $2\hbar\omega_E$ . It turns out  $E'_A = E'_B = E_A$ ; this result confirms that the two atoms exercise spontaneous emission independently and is evidently consistent with the independent-transition assumption.

From the definition of  $|\psi_3\rangle$  in equation (8), it is a straightforward matter to find that the expectation value of the Green function  $1/(z - H)$  in this state is obtained when the relations in equations (25) and (26) are first summed and then projected on the state. The expectation value of the Green function in  $|\psi_4\rangle$  can also be found, following a similar procedure. Specific calculation shows

$$\langle\psi_{3(4)}|\frac{1}{z-H}|\psi_{3(4)}\rangle = \frac{1}{2}\left(\frac{1}{z-2\hbar\omega_G} + \frac{1}{z-2\hbar\omega_E - E'_A - E'_B}\right). \quad (27)$$

The preceding expression is substituted in equation (5) to yield, with the help of an application of the residue theorem, the probability amplitudes for the system to be in states  $|\psi_3\rangle$  and  $|\psi_4\rangle$  at time  $t$ , which, after the modulus square is taken, reduce to

$$P_3 = P_4 = \frac{1}{4}[1 + 2e^{-t\Gamma_0} \cos(2bt) + e^{-2t\Gamma_0}], \quad (28)$$

where  $b = 2\omega_0 + 2\text{Re}(E_A)/\hbar$ . Like the expression in equation (24), the relation in equation (28) states that the decoherence of the system from  $|\psi_3\rangle$  and  $|\psi_4\rangle$  is caused by spontaneous emission of the atoms and follows a nonexponential law too (see the cosine term in the latter equation); but here the explanation to this nonexponential behaviour is different: the cosine term comes from the interference between the probability amplitudes for the system to stay in  $|G\rangle \otimes |G\rangle \otimes |0\rangle$  and  $|E\rangle \otimes |E\rangle \otimes |0\rangle$ . Note the approximation of independent transition in this section exempts the atoms' excited state from any further modification other than that already caused by the radiation reactions.

If the system is initially in states  $|\psi_3\rangle$  and  $|\psi_4\rangle$ , the probability of finding the system in state  $|G\rangle \otimes |G\rangle \otimes |0\rangle$  never changes with time; see equation (25). Thus, unlike  $P_1$  and  $P_2$ , the probabilities  $P_3$  and  $P_4$  settle into a nonzero value  $1/4$  as  $t \rightarrow \infty$ . Besides, since  $|b| \gg \Gamma_0$ , and since  $2|E_{AB}|/\hbar \ll \Gamma_0$  due to the restriction  $R\omega_0/c \gg 1$ , the probabilities  $P_3$  and  $P_4$  oscillate with time more rapidly than  $P_1$  and  $P_2$  do and have an effectively larger decay rate. In [15], an experimental observation also indicates that  $|\psi_3\rangle$  and  $|\psi_4\rangle$  decohere more easily than  $|\psi_1\rangle$  and  $|\psi_2\rangle$ . The reason that  $|\psi_1\rangle$  and  $|\psi_2\rangle$  decohere at a relatively mild rate can be understood like this: the atom–atom interaction (through the vacuum fields) required by the system to evolve when it starts from  $|\psi_1\rangle$  and  $|\psi_2\rangle$  tends to bring the system back to these states after the system spontaneously decays out of them and, thus, makes it relatively difficult for the system to leave these states. In fact, it has been shown that, under certain conditions, such as that the atoms are directly coupled to each other through the dipole–dipole interaction,  $|\psi_1\rangle$  and  $|\psi_2\rangle$  can even become robust states whose entanglement remain unchanged [6].

Although, in the discussion presented in this and preceding sections, decoherence is found to be driven by spontaneous emission of the atoms, it is interesting to note that spontaneous emission can also induce entanglement; see [12] and the discussion in section 2. Thus, entanglement and decoherence are in fact two competing processes.

#### 4. Conclusion

In this paper, the decoherence of two atoms initially entangled in the four Bell states is discussed in the Schrödinger picture. After the Green function is decomposed into a series of atomic and field operators, every order of the atomic transitions that eventually lead to the evolution of the system is considered. The states the system evolves into from the initial states are then found, from which the probabilities the system remains in its initial states are obtained. These probabilities show that the decoherence of the atoms does not follow an exponential law, even though the decoherence itself is revealed to be caused by spontaneous emission. The reason is in calculating the probabilities quantum interference also takes effect.

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